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A DECISION-THEORETIC APPROACH TO ARMS COMPETITION

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ABSTRACT

This paper develops a decision/game-theoretic model of arms growth which implies that arms competitions develop only between nations with conflicting foreign policy goals, and that between such nations arms races between weapons with conflicitng policy missions are ubiquitous. Further implications and extensions of the model are discussed, as are possible avenues of estimation.

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INTRODUCTION

Politicians and scholars alike have expressed concern over the omnipresent lethal stockpiling of weapons by the two superpowers. Few, however, have come to realize that arms growth and arms control are inexorably and fundamentally linked, and that the solutions to arms control lie within the causes of arms growth. As such, this paper focuses upon arms growth. It departs in two respects from the previous literature. First, rather than being simply descriptive, the study is based upon a decision-theoretic model which makes explicit the relation of arms growth to the achievement of mutually incompatible policy goals rather than as "countervalue" competitions in particular weapons systems.[1]

Much research concerning arms races has centered upon the pioneering work of Lewis F. Richardson (1960).[2] The Richardson formulation is well known and will not be discussed in detail herein. The model formulated by Richardson is an excellent descriptive model outlining much of our intuition into the causes and development of arms competitions. The well known differential equations model

$$\begin{aligned} dx/dt &= ky + ax + g \\ dy/dt &= lx + by + h \\ l,\ k &> 0 \\ a,\ b &< 0 \end{aligned} \qquad (1)$$

describes arms races as a competition between two mutually distrustful nations, wherein military budget appropriations or military buildups by one nation are answered in-kind by the competing nation(s). These competitive increases continue indefinitely, abated only by the wealth limits of the competing nations, or by war.

The most plausible operationalization of the Richardson model, the difference equation format, describes nations X's and nation Y's stock of weapons (or military budgets) at time t (X_t and Y_t respectively) as a function of their own previous stock of weapons (X_{t-1} and Y_{t-1} respectively) as well as a function of their adversary's previous stock of weapons (Y_{t-1} and X_{t-1});

$$\begin{aligned} Xt &= \alpha X_{t-1} + \beta Y_{t-1} + c \\ Yt &= \delta X_{t-1} + \gamma Y_{t-1} + f \\ \beta,\ \delta &> 0 \\ \alpha,\ \gamma &< 0. \end{aligned} \qquad (2)$$

But in estimating this simple Richardson interaction model, it is not clear what X and Y should stand for in the arms race context. Richardson thought them to be measures of the "total armed might" of the two mutually distrustful countries, and later tested the model with yearly defense budgets as proxies for X and Y. Most subsequent analysis has also employed defense budgets in this context; Chatterjee (1974), Lambelet (1976), Ruloff (1975), Taagepera et al (1975), and Wagner et al (1975). The model has further been applied to modern treatments of armament races in the missile age; Burns (1959), Boulding (1962), Brito and Intriligator (1977), Intriligator and Brito (1976), Luterbacher (1976), Kent (1963), McGuire (1965, 1977), Pitman (1969), Saaty (1968), and Taagepera (1976), (though many of these later studies applied the Richardson model to stocks of weapons rather than to defense budgets).

The interpretation of defense budgets as a nation's "total armed might" is not entirely unreasonable, as increases in military budgets necessarily precede increases in "total armed might." However, as evidenced in many of the above listed studies, as well as by the low correlation between Soviet and American defense budgets (r = .335) in Figure 1[3], such a proxy gives no indication of the putative arms competition between the two superpowers.

[Figure 1 here]

As indicated earlier, other scholars have pursued the Richardson process through an examination of the stocks of weapons possessed by both sides of a competing pair of nations. Such an analysis, it was thought, might capture the subtle year-to-year changes in armaments which we expect to observe. However, such approaches were often merely a misapplication of disaggregated data -- a misapplication brought about by the poor conceptualization inherent in the Richardson model. Such armament studies have frequently centered upon countervalue competitions as exemplified in Figure 2. Figure 2 depicts Soviet and American stocks of manned strategic bombers, and captures the perceived decrease in American and Soviet bomber strength. However, as is true of many of the earlier mentioned disaggregative studies it exemplifies, no arms competition is evidenced in Figure 2.

[Figure 2 here]

Stated most boldly, the conceptualization problem inherent in the Richardson formulation derives from the fact that the Richardson model does not explicitly take into account the policy characteristics of weapons systems. Every weapons system, whether it be a Marine Corps infantry battalion or a MX missile squadron, has a policy mission for which it was designed and produced to fulfill. To be sure, such systems are often multi-purposed, but the recognition of such policy missions is central to understanding and defining arms competitions. The Richardson formulation by not explicitly considering these policy characteristics of weapons systems is unable to discriminate between which groups of weapons we should (and should not) expect to observe in competition.

The failure to identify Richardson arms races in the aforementioned studies or in Figures 1 and 2 is owed in part to this conceptual shortcoming. The approach to which this paper is addressed, and to which we now turn, the <u>decision-theoretic</u> approach, does incorporate the policy characteristics of weapons into the resulting theory of arms competition. As such the model developed defines not only the shape of the expected arms races, but further predicts between what types of weapons systems we should (and should not) observe competition.

A DECISION-THEORETIC APPROACH TO ARMS COMPETITION

The "total armed might" of a nation is a direct extension of that nation's foreign policy objectives and its overall strategic doctrine. These foreign policy objectives dictate the size and shape of the military force a nation will develop.

A nation's strategic doctine indentifies the types of responses, missions, and/or tasks its military force must be designed to fulfill. Each weapons system procured then fulfills a specific policy mission as necessitated by the needs related to the nation's strategic doctrine.

For example, an American foreign policy objective is the prevention of nuclear conflict. A strategic doctrine developed relative to this objective is mutual deterrence. Specific weapons developed to fulfill policy missions under this doctrine are land-based ICBM's, manned strategic bombers, and sea-based SLBM's each of which has as a policy mission that of inflicting (or threatening to inflict) a nuclear strike on point-targets.

Nations derive political gain[4] from the use, or potential use, of their "total armed might," in accordance with their strategic doctrine. The decision-theoretic approach holds that military force-level decision makers will attempt to maximize this political gain through their choice of weapons, subject to their nation's doctrinal, production, budgetary, and technological constraints.

The basic behavioral postulate to be put forth here about military decision making is that military decision makers select weapons systems and procure armaments in such a manner as to maximize their capability to pursue their nation's foreign policy goals. Given this behavioral assertion, we wish to define a set of refutable hypotheses relating arms race behavior and arms control to the decision calculus just mentioned.

More formally, the behavioral assertion we shall investigate is that a military decision maker engages in some sort of constrained maximizing behavior, the objective of which (for the two nation case A and B) is to maximize

$$\Pi^A(q_1,\ldots,q_n,x_1,x_2,w_1,\ldots,w_n) \qquad (3)$$

where $q_1,\ldots,q_n$ represent weapons allocations for country A, given a set of specific foreign policy goals; x_1, and x_2 represent inputs to the production of the above weapons systems, $w_1,\ldots,w_n$ represent the weapons allocations chosen by an adversary country B, given its own set of foreign policy goals, and $\Pi^A(-)$ represents the decision maker's political gain or profit from deploying $q_1,\ldots,q_n$. This maximization is subject to the production and technology constraints inherent in nation A's economy which we will summarize as

$$\text{production constraint} \quad F(q_1,\ldots,q_n,x_1,x_2) = 0 \qquad (4)$$

Thus we postulate each nation maximizes its own political gain, $\Pi(-)$, by selecting in an optimal fashion the deployment levels for each weapons system in its choice set, $q_1,\ldots,q_n$, and the employment levels of productive inputs to armament manufacture, x_1 and x_2.

The decision-theoretic approach outlined herein does not assume or depend upon any formulation of governmental behavior. The model is consistent with, or at least not inconsistent with, the rational actor, bureaucratic or organizational frameworks developed by Allison (1971). To be sure, the decision calculus is most readily appreciated as a two-person (nation) model and as such fulfills a rational actor framework of government. However, the interactions of various bureaucracies, or the consequences of standard-operating-procedures can result in actions which altogether appear as if the bureaucracy or organization was acting to maximize political gain as asserted.

We will assume that equation 3 can be strictly linearized, in that the total overall foreign policy profit for a military program $\Pi^A(-)$ can be represented by a linear sum of the political gains derived from the capabilities of the individual weapons systems to perform their specific policy tasks. Thus we will assert that decision makers act to

$$\text{maximize} \quad \Pi^A(q_1,\ldots,q_n,x_1,x_2,w_1,\ldots,w_n) = \sum_j \Pi_j^A(q_1,\ldots,q_n,x_1,x_2,w_1,\ldots,w_n) \tag{5}$$

$$\text{subject to} \quad F(q_1,\ldots,q_n,x_1,x_2) = 0$$

where Π_j^A represents the individual foreign policy profits for the j^{th} weapons system.

This maximization is performed with respect to q and x, the armament levels deployed and the production inputs employed, given their expectations with regards to their opponent's reactions.

Further, assuming continuity for the political gain function, Π, and the production function, F, and given the following set of assumptions an equilibrium level of arms exists for each weapons sytem, and thus the decision-theoretic model is also a game-theoretic formulation. Before we derive the theorems relating this decision structure to observable artifacts of arms races we need to make explicit the several interactive and behavioral assumptions arising from the arms race context. First, the political profit function is an increasing function of quantity of one's own armaments and a decreasing function of production inputs, formally

$$\begin{aligned} \partial\Pi_k^A/\partial q_j &> 0 \qquad \text{for all } k, j \\ \partial\Pi_k^A/\partial x_i &< 0 \qquad \text{for all } k, i \end{aligned} \tag{6}$$

Thus, increasing the level of armaments deployed, all else constant, leads to an increase in foreign policy profit, and similarly to a decrease in inputs employed. Second, the usual production assumption is that production is an increasing function of productive inputs, i.e., the marginal productivities of both inputs are positive,

$$\partial F/\partial x_i > 0 \qquad \text{for all } i = 1,2 \tag{7}$$

Further, in line with standard economic theory we will assume that the production constraint is a decreasing function of the quantity of armaments produced, all else constant,

$$\partial F/\partial q_j < 0 \qquad \text{for all } j. \tag{8}$$

Along the lines of the discussion in the second section of the text, we will assume that a change in nation B's j^{th} weapons system (say tanks) will not have an effect on country A's allocative decision or political profits for its i^{th} weapons system (for $i \neq j$) (e.g., aircraft carriers).

$$\partial \Pi_k^A/\partial w_j = 0 \qquad \text{for all } k \neq j \tag{9}$$

This assumption follows from the observation that armament decisions are interrelated between the two adversary nations if and only if the policy goals of the two nations (and in particular the policy tasks to be performed by the weapons systems in question) are mutually exclusive. The specific form of (9) follows from the fact that we can arrange the vectors of armaments for each nation, $q_1,\ldots,q_n$ and $w_1,\ldots,w_m$ in a particular order such that the i^{th} weapon system in each arsenal is the counterforce option of each of the respective nations, i.e., we can arrange (order) the weapons systems in such a fashion as to put weapons systems for one nation against the corresponding weapons system for the other nation which has a mutually exclusive policy goal.

We will further specify the following conjectural variation, that is, the interactive assumptions that we postulate the decision

maker adheres to,

$$\begin{aligned} &\partial w_k/\partial x_j = 0 \qquad \text{for all } k, j \\ &\partial w_k/\partial q_j \neq 0 \qquad \text{for all } k, j. \end{aligned} \tag{10}$$

These conjectural variations specify that a change in the level of productive inputs employed by country A for the production of its arsenal has no effect on the level of armaments procured by country B, and that the decision makers believe that an increase in their own armament levels will not go unnoticed by their adversary (if and only if the policy tasks of the weapons systems are mutually exclusive).

Assume also that the political profits function is convex with respect to the quantity of armaments procured (this insures unique maximum points) and concave with respect to production inputs employed,

$$\begin{aligned} &\partial^2 \Pi_k/\partial q_j^2 < 0 \qquad \text{for all } k,j \\ &\partial^2 \Pi_k/\partial x_i^2 > 0 \qquad \text{for all } k,i \end{aligned} \tag{11}$$

Combined with assumption (6) this gives a generalized shape for the political profit function which insures unique, nontrivial maximum points. These two assumptions together, (6) and (11), specify that the political gain accrued from the allocation of a specific weapons system (all else constant) is increasing at a decreasing rate, i.e., the gain in political profits from the first allocation of the weapons system is greater than the gain obtained from later stockpiling of such weapons systems.

Similarly, the assumptions relate that the marginal political productivity of each productive input is negative (ceteris paribus) and decreasing, i.e. that the loss in political profits from hiring an extra unit of productive input (ceteris paribus) increases as more inputs are hired.

Three further assumptions are necessary to derive unique maximization points for (5) and to proceed with the development of meaningful (and testable) hypotheses about arms race behavior. First is the standard assumption that the second order sufficiency conditions for a constrained maximization are fulfilled. This I will give without justification in the following form: that the determinant of the bordered Hessian matrix of second order determinants (to be discussed in (16) is positive. Second are the usual assumptions as to the shape of the production function, again without justification,

$$\partial^2 F/\partial x_i \partial q_j = \partial^2 F/\partial q_j \partial x_i = 0 \qquad \text{for all } i,j \tag{12}$$

$$\partial^2 F/\partial x_i \partial x_j = \begin{cases} \leq 0 & i=j \\ > 0 & i \neq j \end{cases}$$

Third, we will assume that the political profits function given in (5) is a general linear function of production inputs and armament outputs. This assumption as well takes the form of specific constraints upon the second order partial derivatives, in this case,

$$\partial^2 \Pi_k^A/\partial q_j \partial x_i = \partial^2 \Pi_k^A/\partial x_i \partial q_j = 0 \qquad \text{for all } k,i,j \tag{13}$$

$$\partial^2 \Pi_k^A/\partial x_i \partial x_j = 0 \qquad \text{for all } k,\ i \neq j$$

The general linear form specifies only that productive inputs and armament outputs do not interact in any multiplicative fashion in the political profits function: that the effects of each on political gains is separable. A necessary consequence of this behavior is that the first order partial derivatives of the following lagrangian equal zero:

$$L = \sum_j \Pi_j + \lambda F \tag{14}$$

where λ is the Lagrange multiplier. Hence, given our interaction assumption $\partial \Pi_k^A/\partial w_j = 0$ as well as our conjectural variation $\partial w_k/\partial x_{ij} = 0$, we can summarize the model thus far as

maximize $\qquad \Pi_j(q_j, x_1, x_2, w_1, \ldots, w_n) \quad j = 1,\ldots,n$

where $\qquad x_i = (x_i, \ldots, x_{ij}, \ldots, x_{in}) \qquad i = 1,2$

subject to $\qquad F(j) = f_j(x_1, x_2) - q_j = 0 \qquad j = 1,\ldots,n$

$$L = \Pi_j + \lambda F(j)$$

$$\begin{aligned} \partial L/\partial q_j &= \partial \Pi_j/\partial q_j + \partial \Pi_j/\partial x_1 \cdot \partial x_1/\partial q_j + \partial \Pi_j/\partial x_2 \cdot \partial x_2/\partial q_j \\ &\quad + \partial \Pi_j/\partial w_j \cdot \partial w_j/\partial q_j + \lambda \partial F(j)/\partial q_j = L_j = 0 \\ \partial L/\partial x_i &= \partial \Pi_j/\partial q_j \cdot \partial q_j/\partial x_i + \sum_k^2 \partial \Pi_j/\partial x_k \cdot \partial x_k/\partial x_i + \lambda \partial F(j)/\partial x_i \\ &= L_{x_i} = 0 \\ \partial L/\partial \lambda &= F(j) = L_\lambda = 0, \end{aligned} \tag{15}$$

since from our additivity assumptions the maximization of the sum of political profits is equal to the sum of the maximization of individual profits from each weapons system, and thus maximization of $\Pi_j^A(-)$ for all j simultaneously is equivalent to the maximization of $\Pi^A(-)$. The expressions on the right hand side of (15)' represent just a notational convenience for the expressions of the derivatives on the left hand side.

Further, for expository ease, in the equations which follow, denote

$$L_{jx_i} = \partial^2L/\partial q_j\,\partial x_i, L_{jj} = \partial^2L/\partial q_i^2, Lj_{w_k} = \partial^2L/\partial q_j \partial w_k \tag{15}$$

$$F_j = \partial F/\partial q_j,\text{ and } F_{xi} = \partial F/\partial X_i.$$

Equation (15)' represents the <u>necessary conditions</u> for a constrained maximum of political gain (from weapons procurements), <u>sufficient conditions</u> for such a maximum have previously been outlined, that the determinant of the matrix of second order partial derivatives be positive.

$$H = \begin{vmatrix} L_{jj} & L_{jx_1} & L_{jx_2} & F_j \\ L_{x_1j} & L_{x_1x_1} & L_{x_1x_2} & F_{x_1} \\ L_{x_2j} & L_{x_1x_2} & L_{x_2x_2} & F_{x_2} \\ F_j & F_{x_1} & F_{x_2} & 0 \end{vmatrix} > 0 \tag{16}$$

These conditions will be assumed to be fulfilled.

Partially differentiating the first-order conditions in (15) to derive the second order partials of the lagrangian given in (16) yields,

$$L_{jj} = \Pi_{jj} + \lambda^*F_{jj} = \Pi_{jj} < 0$$

$$L_{x_ix_j} = \Pi_{x_ix_j} + \lambda^*F_{x_ix_j} = \begin{cases} <0 & i=j \\ >0 & i\neq j \end{cases}$$

$$L_{jx_i} = \Pi_{jx_i} + \lambda^*F_{jx_i} = L_{x_ij} = \Pi_{x_ij} + \lambda^*F_{x_ij} = 0 \tag{17}$$

$$L_{\lambda j} = F_j = 0$$

$$L_{jw_k} = \Pi_{jw_k} + \lambda^*F_{jw_k} = \Pi_{jw_k} \begin{cases} < 0 & \text{if } j = k \\ = 0 & \text{if } j \neq k \end{cases}$$

$$L_{x_iw_j} = 0$$

Note, the sign of L_{jw_k} (< 0) follows directly from the assumption that $\partial^2\Pi/\partial q_j \partial w_k < 0$, if j = k.

In matrix notation the above system of equations appears as follows:

$$\begin{bmatrix} L_{jx_1} & L_{jx_2} & F_j \\ L_{x_1x_1} & L_{x_1x_2} & F_{x_1} \\ L_{x_1x_2} & L_{x_2x_2} & F_{x_2} \\ F_{x_1} & F_{x_2} & 0 \end{bmatrix} \begin{bmatrix} \partial q_j/\partial w_j \\ \partial x_1/\partial w_j \\ \partial x_2/\partial w_j \\ \partial\lambda/\partial w_j \end{bmatrix} = \begin{bmatrix} -L_{jw_1} & \cdots & -L_{jw_j} & \cdots & -L_{jw_m} \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \tag{18}$$

The above sets of equations represent merely the steps necessary to define a unique, non-trivial maximum of political profits and to establish the set of refutable hypotheses about to be discussed. A unique maximum of political profits has been assured by the assumptions as to the shapes of the production-technology constraint and political profits function given in the previous section. Similarly the testable hypotheses about arms competition follow directly from the maximization procedure outlined here and from the assumptions given previously.

Given the matrix notation for the system of second-order partials we can solve for $\frac{\partial q_j}{\partial w_k}$, the rate of change of country A's armament levels corresponding to a change in the adversary nation B's armament level as a function of the rates of change of the production and political profits function. Solving for $\frac{\partial q_j}{\partial w_k}$ by Cramer's rule,

$$\partial q_j/\partial w_k = \underbrace{\begin{vmatrix} -L_{jw_k} & L_{jx_1} & L_{jx_2} & F_j \\ 0 & L_{x_1x_1} & L_{x_1x_2} & F_{x_1} \\ 0 & L_{x_1x_2} & L_{x_2x_2} & F_{x_2} \\ 0 & F_{x_1} & F_{x_2} & 0 \end{vmatrix}}_{H} \tag{19}$$

$$= \frac{-L_{jw_k} H_{11}}{H}$$

$$\text{where } H_{11} = \begin{vmatrix} L_{x_1x_1} & L_{x_1x_2} & F_{x_1} \\ L_{x_1x_2} & L_{x_2x_2} & F_{x_2} \\ F_{x_1} & F_{x_2} & 0 \end{vmatrix} \tag{20}$$

where H is the aforementioned bordered Hessian determinant of the coefficients matrix.

The partial derivative has a predictable sign. Since $H > 0$, and $H_{11} > 0$ by the second order conditions, and since $L_{jw_k} < 0$, we have

$$\partial q_j/\partial w_k = -L_{jw_k} H_{11}/H \begin{cases} > 0 \text{ if } j = k \\ = 0 \text{ if } j \neq k \end{cases} \quad \text{(RH2)} \tag{21}$$

A multi-party extension of the result derived above is easily developed employing similar assumptions and yielding similar results. The basic behavior postulate for the multi-party decision calculus would be that military decision makers act to maximize

$$\Pi(q_1,\ldots,q_n,x_1,x_2,w_1^1,\ldots,w_n^1,\ldots,w_1^K,\ldots,w_n^K) = \sum_j \Pi_j(q_1,\ldots,q_n,x_1,x_2,w_1^1,\ldots,w_n^K) \tag{22}$$

subject to $F(g,x) = 0$, where q, x, F, and Π are as before and w_j^i represents th4 j^{th} weapons system for the i^{th} country. The comparative statics results would be as follows:

$$\partial q_j / \partial w_k^\ell = \begin{cases} > 0 & \text{if the foreign policy goals of nation A and } \ell \text{ are conflicting and } j = k \\ = 0 & j \neq K, \ \forall \ell \\ \leq 0 & \text{If the foreign policy goals of nation A and } \ell \text{ are compatible and } j = K. \end{cases} \tag{23}$$

Thus we postulate that military decision maker chooses the level of weapons deployments, q_i, given their expectations for the level of weapons deployments for their adversaries, and the level of complimentary weapons deployments for their allies. Equation 23 relates that the military decision makers 1) increase the level weapon(s), q_i, in reaction to an increase in the level of weapon(s) w_j^ℓ by their competitor; 2) disregard, in the allocation of q_j, changes in the level of w_k^ℓ ($j \neq k$) for either allies or adversaries and; 3) apply some portion of their allies weapons stock w_j^ℓ which fulfill the same policy tasks as q_j to their political profit maximization.

Thus, alliances enable nations with compatible foreign policy objectives to take advantages of differential technologies across nations. For example, if the Americans are more capable of producing aircraft, while the British are better tank builders, then the Americans will specialize in aircraft (and built some tanks), while the British will deploy mostly tanks, and both will count some of the others weapons stocks as fulfilling their needs in relation to their foreign policy objectives.

Equations 21 and 23 represent the arms race hypothesis as derived from the decision-theoretic model. Equation 23 translates into our first refutable hypothesis (RH1),

RH1 *Arms races develop <u>only</u> between nations with conflicting foreigh policy goals.*

Whereas, equation 21 translates into our second refutable hypothesis (RH2),

RH2 *Arms competitions develop <u>only</u> between weapons systems with conflicting policy missions (i.e., only between counterforce weapons groups).*

DISCUSSION

The theory of arms competition just outlined is unique in that the roots of the arms competition are explicitly decision-theoretic. The policy goals of the arming nations and the policy aspects of the specific armaments are explicitly imbedded in the decision framework. As such, the decision analysis explicitly defines the type of arms competition, i.e., competition between counterforce groups, which we expect to observe. Arms competition, as shown by (3) through (23), arises from decentralized (nationalistic) decision-making by decision-makers acting to maximize political (foreign policy) gain subject to constraint.

In contrast the Richardson model (and its hybrids), by divorcing the policy aspects of weapons from their armament characteristics, is unable to define a priori where we should expect to observe arms competition. Further, arms competition in the Richardson framework arises as by a "law of nature," in a mechanistic fashion, divorcing decisions made by policy makers from armament levels.

The two hypotheses, RH1 and RH2, make explicit what Richardson did not, and conceptualize arms competitions in an estimatable fashion. These propositions further allow us to comment on the tenor of the current national armament debate, in that most academics and politicians who deplore the arms race speak in countervalue terms, and accuse administrations and military decision makers of a maniacal desire to destroy the world many times over. The decision-theoretic model and the resulting propositions suggest that this situation of immense countervalue redundancy emerges epiphenomenally from a rational counterforce decision calculus which is quite cost-conscious.

Figures 3 - 5 represent a brief analysis of the second refutable hypothesis derived above.[5] The figures represent time-series of counterforce weapons inventories for the U.S. and Soviet Union. The high (and positive) correlations between the counter-force groups depicted give preliminary evidence to the existence of counterforce arms competitions.

[Figures 3, 4, 5 here]

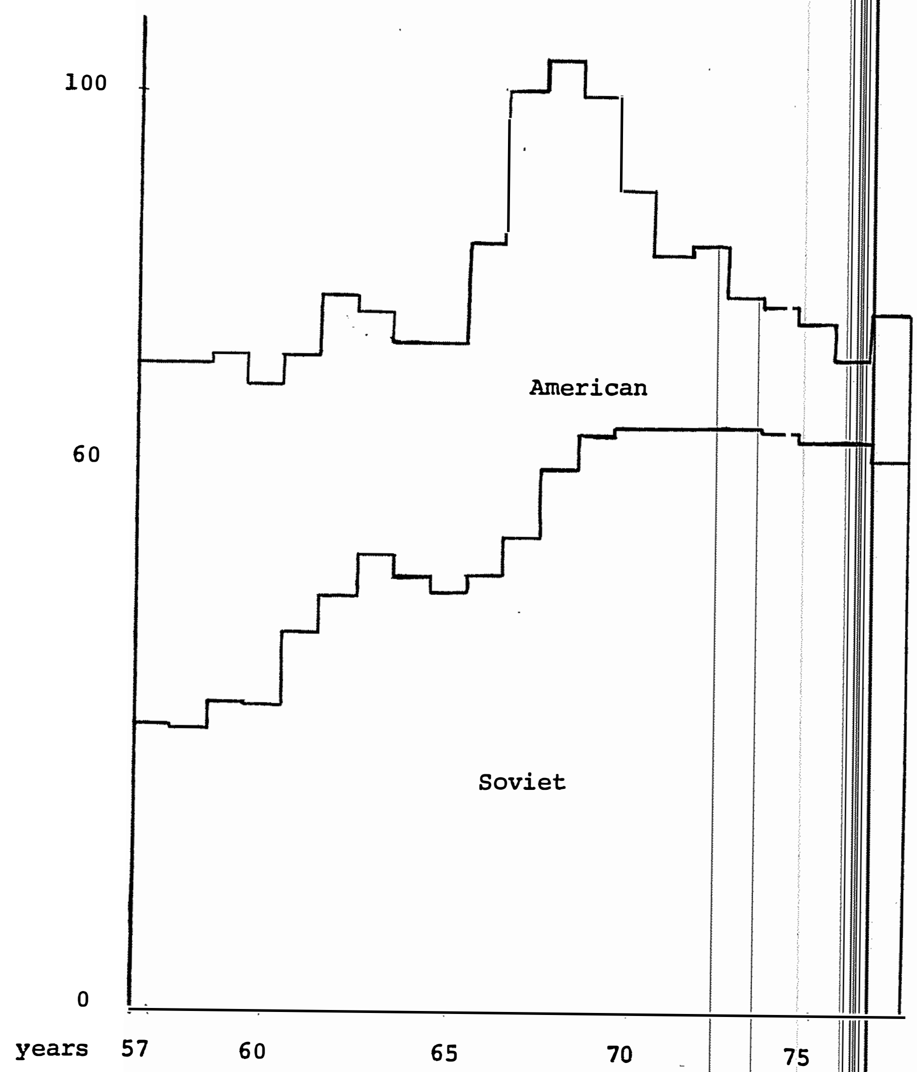
The model develops two refutable hypotheses in an estimatable fashion. Bilateral and multilateral tests of RH1 and RH2 can be derived employing time-series force-level data as depicted in Figures 3 - 5. Further, insights into current arms control agreements, their strengths and limitations, can be derived from the decision-theoretic analysis.

Figure One

American-Soviet Defense Budgets

Constant 1973 Dollars

(in billions)



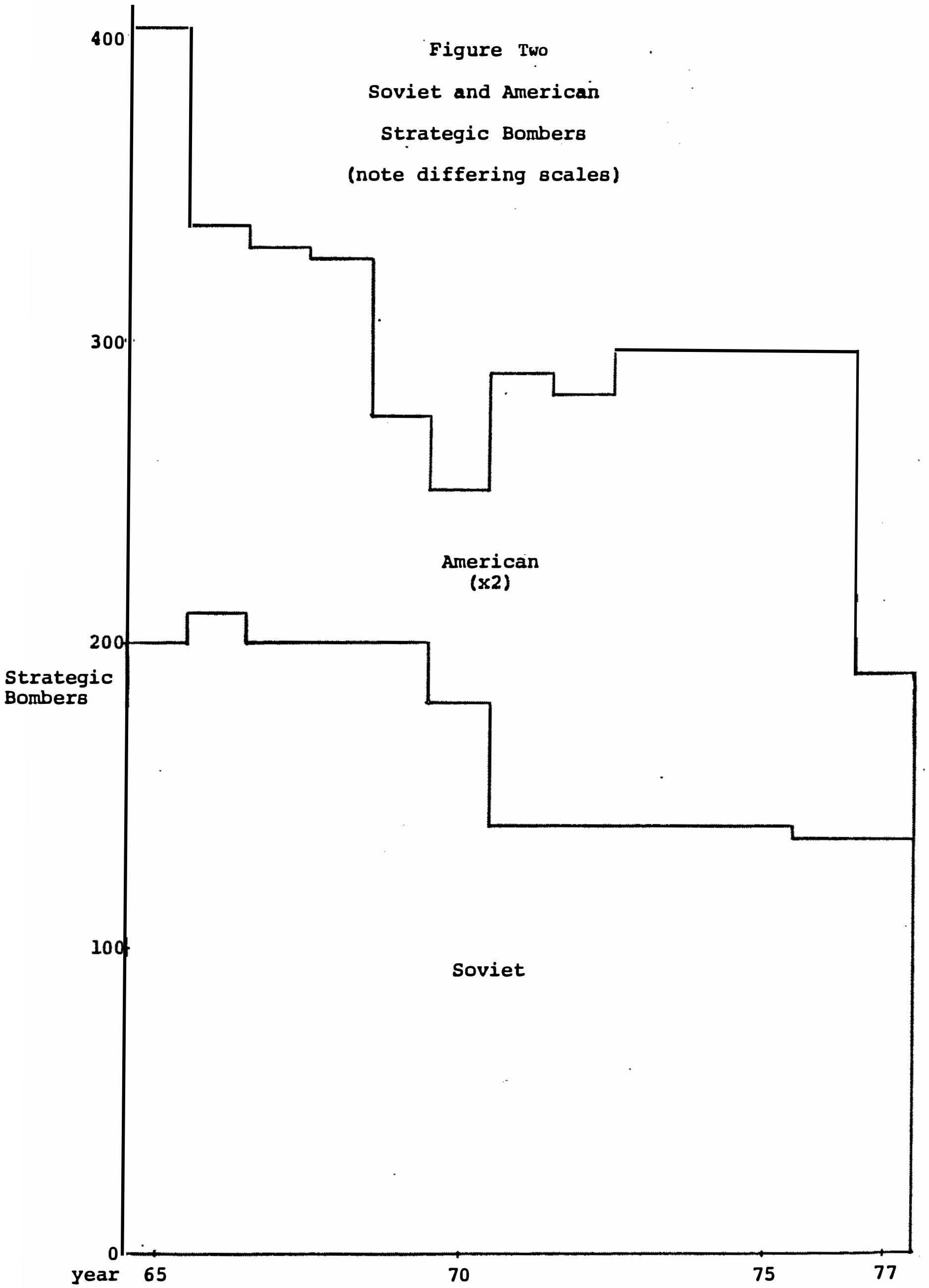


Figure Two
Soviet and American
Strategic Bombers
(note differing scales)

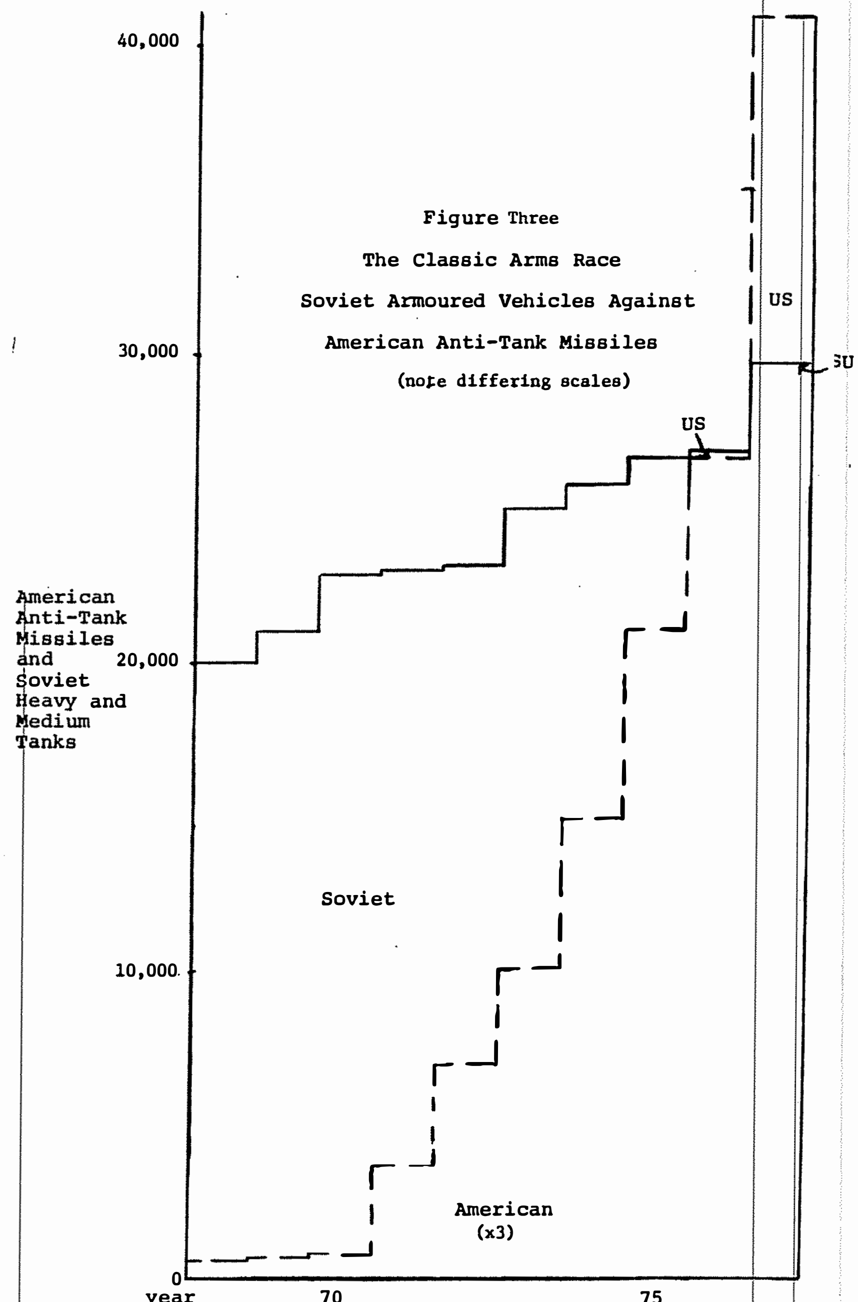


Figure Three
The Classic Arms Race
Soviet Armoured Vehicles Against
American Anti-Tank Missiles
(note differing scales)

Figure Four

The Classic Disarmaments Race I

Soviet Interceptors Against American Bombers

(note differing scales)

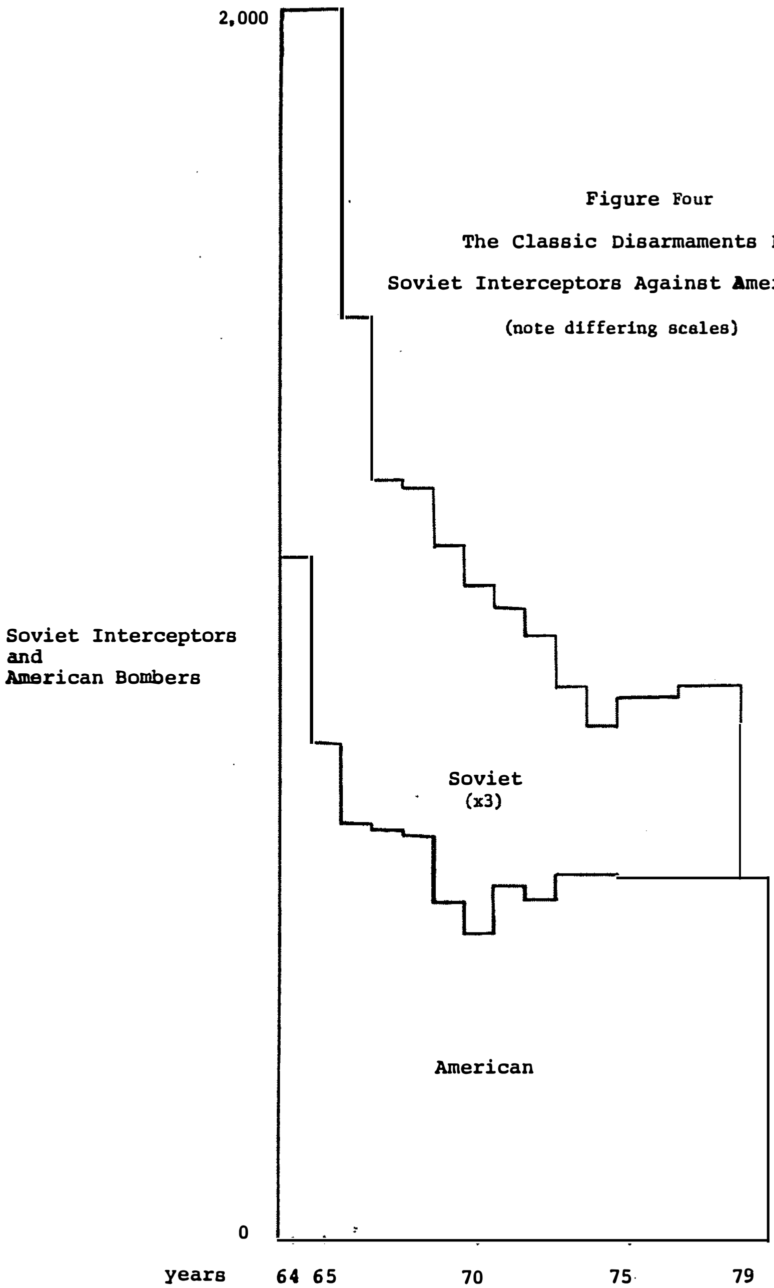
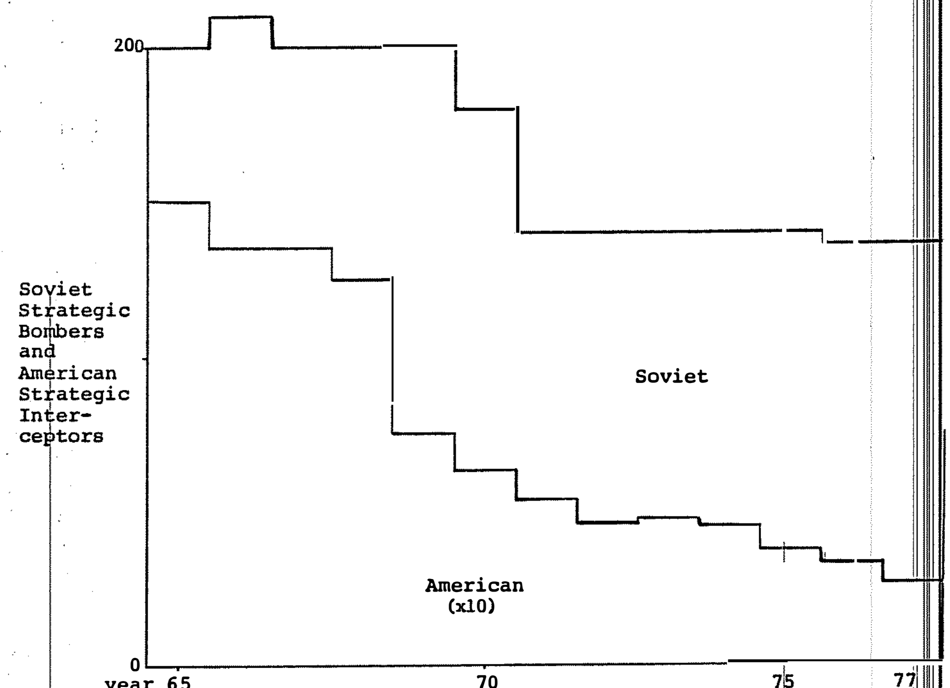


Figure Five

The Classic Disarmaments Race II

Soviet Strategic Bombers Against American Strategic Interceptors

(note differing scales)



FOOTNOTES

* The author wishes to thank Bob Forsythe, Ed Green and Louie Wilde for many helpful comments.

1. Counterforce in this context refers to pairs of weapons systems with conflicting policy missions (e.g., bombers and interceptors), while countervalue in this context refers to pairs of weapons systems with similar policy missions (e.g., bombers and bombers). Further, arms competitions through-out this analysis refer to disaggregated individual competitions as opposed to the interpretation prevailing in the literature that arms races are aggregate overall phenomena.

2. The basic Richardson model was explained and given wide circulation by Rapoport (1957, 1960). Stability conditions were discussed by Chase (1969), a multi-nation version based on Richardson has been developed by O'Neil (1970). An armament game devised by Friberg and Jonsson (1968) was observed to lead to mutual arms escalation. Caspary (1967) critiqued the Richardson framework and proposed a rather complex alternative. The relation between arms race and war initiation has been investigated by Intriligator (1964).

3. The budget estimates in Figure 1 are S.I.P.R.I. estimates, and are employed as many of the arms race studies listed employed

S.I.P.R.I. budget estimates in their analysis. The figure also serves to demonstrate the problem implicit when employing budgets as a proxy for armed might, as budget estimates for military expenditure tend to differ dramatically from one source to another, especially estimates for nonmarket economies as the Soviet economy.

4. The political gain (or profit) a nation derives from the deployment of a specific weapons system can consist of a combination of foreign policy and domestic political gain. No assumption is made concerning the content of political gain, as it is employed merely to represent the returns a nation receives from weapons deployment.

5. The data for figures 2 - 5 is from Collins (1978) and the Military Balance (1964-1978). In figure 3, Soviet armoured vehicles include medium and heavy tanks plus armoured fighting vehicles and armoured personnel carriers. Figures 4 and 5 depict the number of strategic (nuclear) bombers and interceptors (as opposed to tactical bombers and interceptors) as specified by the Department of Defense.

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